Mathematical Trace of Dimensional Hierarchy in the World Heritage Selimiye Mosque*

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ABSTRACT
The Selimiye Mosque and its social complex, which was accepted into the World Heritage List on June, 29 2011, carry cultural, locational and structural characteristics that are important for the world of architecture. The Selimiye Mosque reflects political power, abundance of the Ottoman Empire and the era, in which it was built, as well as genius approach of its designer, which pushes the limits of construction, with its religious and social meanings. The purpose of this study is to convert traces of aesthetic achievement created by dimensional hierarchy, which was designed by Architect Sinan on structural configuration of a long-span construction like the Selimiye Mosque, into mathematical data. First of all, method and limitations are described in this study and contributions of study results to scientific environment are stated. Then, admission process of the Selimiye Mosque through the concept of the World Heritage is touched upon and its architectural properties are addressed. Hierarchy, which were determined on longitudinal section of the mosque, were measured in order to query mathematical trace of dimensional hierarchy and the data obtained were analyzed via drawn function graph. Thus, functions regarding hierarchy of outer shell are detected and the formula of dimensional hierarchy is presented to scientific environment. Consequentially, it was determined that dimensional hierarchy, which was configured by Architect Sinan in the Selimiye Mosque and supports immense internal space, has a curvilinear (polynomial) character. Therefore, it was observed that a structurally balanced and an aesthetic transition formed for reaching from human scale to monumental building in the mosque's outer structural configuration.

Keywords: Dimensional hierarchy, Mathematical trace, Selimiye Mosque, Architect Sinan

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1. INTRODUCTION

"The main composition problem of outer structural configuration of long-gauge religious buildings in monumental character (such as Selimiye, Haghia Sophia, St.Pietro and Santa Maria DellaFiore) are to not be able to provide spatial continuity in transition from medium-space upper cover to lower structure. Architect Sinan solved this problem with vertical elements that connected horizontal masses to each other in hierarchy at pyramidal elevation. These vertical elements also became instruments for a dynamic mass design" (Kuban, 1997). These structural trials of Architect Sinan reached a point of perfection with dimensional hierarchy on outer structural configuration of the Selimiye Mosque, a work of mastery.

The Selimiye Mosque came into being as a work of creativity by a master architect with the highest construction and architectural approach of its era. While Architect Sinan concretized the power of the Ottoman Empire and supremacy of the Islamic religion, he seized a balanced transition from monumental building scale to human scale in configuration of mass plastic. The famous German architect, Bruno Taut, described the

Selimiye as “The City Crown” when he saw it on Edirne’s skyline (Kuban, 1997). In this regard, the Selimiye Mosque took its place in the history of humanity as a symbol building.

2. THE SELIMIYE MOSQUE AS THE WORLD HERITAGE AND ARCHITECTURAL PROPERTIES

The Selimiye Mosque and its social complex were accepted in the World Heritage list on 29 June 2011. The process of admitting the Mosque and its social complex as a world heritage began with notification of addition to Temporary list of World Heritage to UNESCO in 2000. File nominee preparations, which started in 2006, were completed in 2010 and sent to UNESCO. The Selimiye Mosque and its social complex were declared a world heritage with unanimity at the 35th gathering of Committee carried out between 19-29 June 2011. In following sections of study, architectural properties of the Selimiye Mosque as World Heritage will be examined (Turkish National Commission for UNESCO).

Architect Sinan was responsible from all construction works of the Ottoman Empire during his half-century tenure of being a master architect (1538-1588) and built approximately 500 buildings with different functions and types within boundaries of the Empire. Architectural understanding of Sinan has an extensive diversity and flexibility that adapts to conditions of his era. In this respect, his works gained totality within the framework of concepts of human, space, scale, form, meaning, context, aesthetic, environment and symbolism (Bozdoğan et al., 2006)

This monumental building, which was built by an order of the Sultan Selim the Second in Edirne between 1569-1575 and was characterized by Architect Sinan as “my work of mastership”, became one of the most successful religious architectural examples of its era and following eras with its technical perfection, dimensions and aesthetic values. The Selimiye Mosque and its social complex, a symbol of Edirne, is a work of an expert architectural and city planning understanding with its four minarets, which can be perceived from every place of this city, its magnificent dome, its inner court with a Şadırvan (fountain), Dar-ul Kurra and Dar-ul Hadis (madrasahs) that symmetrically encircle this structure towards the wall of qibla, Arasta Bazaar, its infants’ school and with its outer court walls (Günay, 1998; Kısa Ovalı et al., 2016), (Figure 1).

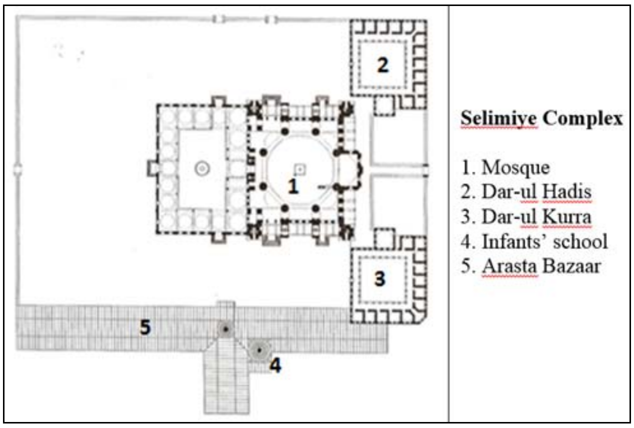

Figure 1. General settlement plan of the Selimiye Mosque and its Social Complex (Kuban, 1997)

Sinan voiced his opinions on his work of mastership, the Selimiye Mosque, in “Tezkiretu’l-Bunyan”, which was penned by his close friend Sai, as follows (T.C. Kültür Varlıkları ve Müzeler Genel Müdürlüğü): “I completed my journeyman training in the Sehzade Mosque in Istanbul. And I completed my mastership in the Süleymaniye Mosque. But, I displayed and explained my expertise by exerting all my power on this Sultan Selim Han Mosque. We made such a great mosque that it is worth all people’s liking in Edirne”.

2.1. SETTLEMENT PLAN

The mosque platform was constructed at the center of the city of Edirne over a hill commanding the city which can be seen with all its majesty from the Süleymaniye Village of Uzunköprü, Rhodope Mountains of Bulgaria and the city of Oresteia in Greece (Figure 2). There are a wide outer court, which encircles the Mosque, Dar-ul Kurra and Dar-ul Hadis Madrasahs that delimit this court and Arasta Bazaar on southwestern direction. Today, Dar-ul Kurra is used as the Foundation Museum of Edirne, Dar-ul Hadis is used as the Museum of Turkish and Islamic Arts and infants' school is used as the Tourism and Promotion Association of Edirne. Arasta Bazaar still sustains its unique function as a touristic market in which handicrafts and palatal delights specific to Edirne are being presented to tourists.

Figure 2. Inner-city location of the Selimiye Mosque (Şengül, E., Personal Archive)

Due to necessity of supporting the mosque platform with a retaining wall on southwestern direction, it is considered that Architect Sinan designed Arasta there however [1]; its construction was achieved by Davut Aga. Thus, a lower structure covered with vaults was established under outer court of the Mosque and Sinan might have converted it to shop sequences. Infants' School and the Praying Dome on Arasta are designs which have maturity that could be constructions by Sinan (Figure 3).

Figure 3. Overall appearance of the Selimiye Mosque and view of the Selimiye Mosque from court of Dar-ul Hadis (Şengül, E., Personal Archive)

Outer court of the Selimiye Mosque was delimited by two exactly same Madrasahs asymmetrically planned according to their entries used as Dar-ul Hadis and Dar-ul Kurra on the side of qibla, court wall to northern and eastern directions and the wall of Arasta to western direction. Five doors including crown gate on qibla axis, two doors over the street of stone rooms, one door over court wall towards qibla and the last one over Arasta wall are used to enter from urban space into outer court.

2.2. SPATIAL AND STRUCTURAL CONFIGURATION

Main dome of the Selimiye Mosque with a diameter of 31.50 meters is elevated with an octagon frame consisting of eight columns situated on an edged-square base. Height of dome is 43.28 meters at center. The dome of the Selimiye covers some 30% of the mosque's floor surface (2000 m^2), (Günay, 1998). There is a two-story surrounding space (ambulatory) shaped by this structure around this unhindered immense space of salaat. Spatial alcove on qibla wall on the plan is an effect of the objective for ensuring symmetry of static system of this octagon. Architect Sinan converted prop elements inside and outside the structure to aesthetic elements of an architectural space and mass organization. Spatial and mass organization of the Mosque gains a unity with a place for worship, narthex, four symmetrically-placed minarets, an inner court with porch and an encircling outer court (Kuban, 1997; Kısa Ovalı et al., 2016), (Figure 4).

Figure 4. Unhindered salaat volume inner space of the Selimiye (Şengül, E., Personal Archive)

Bringing spatial impact in centrally-planned buildings with wide openings and domes to the optimum reveals the following problems in terms of dimension (Kuban, 1997):

a. Establishing balance of volumetric combinations of medium space and surrounding spaces,
b. Ensuring balanced placement of carrying bases and decreasing their bulky appearance.

These problems were solved by Architect Sinan in spatial organization of the Selimiye Mosque and in its structural order as follows:

a. Side spaces, which encircle medium space, were brought closer to boundaries of medium space as much as possible and volumetric transitions were configured in a graded way.
b. Bulky appearances of carrying bases were lessened with niches by keeping their dimensions minimum.

Court of the Selimiye Mosque has specific design properties. The courtyard is very large. Composition of this portico, which is surrounded by 18 domes and 16 columns, displays a rhythm unseen in other mosques. Porticos of narthex were organized with 5 domes carried by 6 columns thicker than other court columns (Kuban, 1997; Günay, 1998), (Figure 5).

Figure 5. Inner courtyard, Last Congregation Ground of Selimiye and Dargah Door (Kısa Ovalı et al., 2016)

Ensuring dimensional balance between court and mosque mass in Ottoman mosque architecture constitutes the primary problem of outer space configuration. Architect Sinan solved structural and spatial balance by configuring relationship between narthex and court porticos within dimensional hierarchy. Narthex porches were emphasized with narrow openings and low arches on both sides of entry opening in a way that will complete structural scheme of octagonal frame. A hierarchy at a dimensional connection to main dome was created by designing narthex domes greater than domes covering court porticos (Kuban, 1997).

3. METHODOLOGY

Aesthetic balance in entirety of mass in the Selimiye Mosque is salient in transition from human scale to monumental building (it can also be looked at as a transition from monumentality to human scale) in the Selimiye Mosque in which organization of internal space and mass was shaped according to adoptions of the Islamic religion. In this study, mathematical trace of dimensional hierarchy, which ensured this balance, is investigated.

For this reason, main grades that represent the beginning and end of mass on longitudinal section of the Selimiye Mosque were measured by using the "TOPCON GTS-229 Total station" range finding device. Measuring method is based on the principle of finding heights of objects, whose distance is unknown or could not be measured, by means of trigonometric measurements. The data (vertical-horizontal distances) obtained as a result of measurements are analyzed via drawn function graph, and presence of a function regarding hierarchy of outer shell is examined.

Structural problems arise, static balances become fragile and a sense of disproportionality in people might occur in cases where human scale is completely ignored and structural gauges are not balanced in architectural designs; in other words where dimensional hierarchy cannot be ensured.

The following gains will be obtained with a dimensional hierarchy study performed on outer structural configuration of the Mosque within the framework of this study:

- Mathematical trace of the Selimiye Mosque in dimensional hierarchy will be usable in aesthetic configuration of mass balance for urban areas which will be newly created (major settlement regions) or complex buildings (in building groups consisted of segmented masses),
- Hierarchy curve regarding the function which will emerge in dimensional hierarchy will have characteristics that will allow revision of master plans or preparation of new master plans in terms of preservation of city skylines.

4. HIERARCHY ANALYSIS IN DIMENSIONAL CONFIGURATION

Architect Sinan solved transition from upper-structure cover to lower structure with vertical elements that connect lateral masses to each other in hierarchy at pyramidal height. These vertical elements are also instruments of a dynamic mass design. These grades forms transition from human scale to monumental building dimension.

Grades, which were used for quest for mathematical formula of dimensional configuration of the Selimiye Mosque, were specified as points that identify primary beginning and ends related to structural construction in configuration of outer structure as follows (Kısa Ovalı et al., 2013), (Figure 6):

1st Grade, portico compositions encircling all three sides of inner court,
2nd Grade, gathering places which form surrounding spaces within the main mass and dome end of narthex in configuration of outer structure,
3rd Grade, gallery flat that supports main dome frame with semi-domes,
4th Grade, octagonal frame that carries the main dome,
5th Grade, upper frame of the main dome,
6th Grade, main dome end that covers medium space,
7th Grade, end point of minarets and qibla wall that complete dimensional hierarchy at pyramidal elevation.

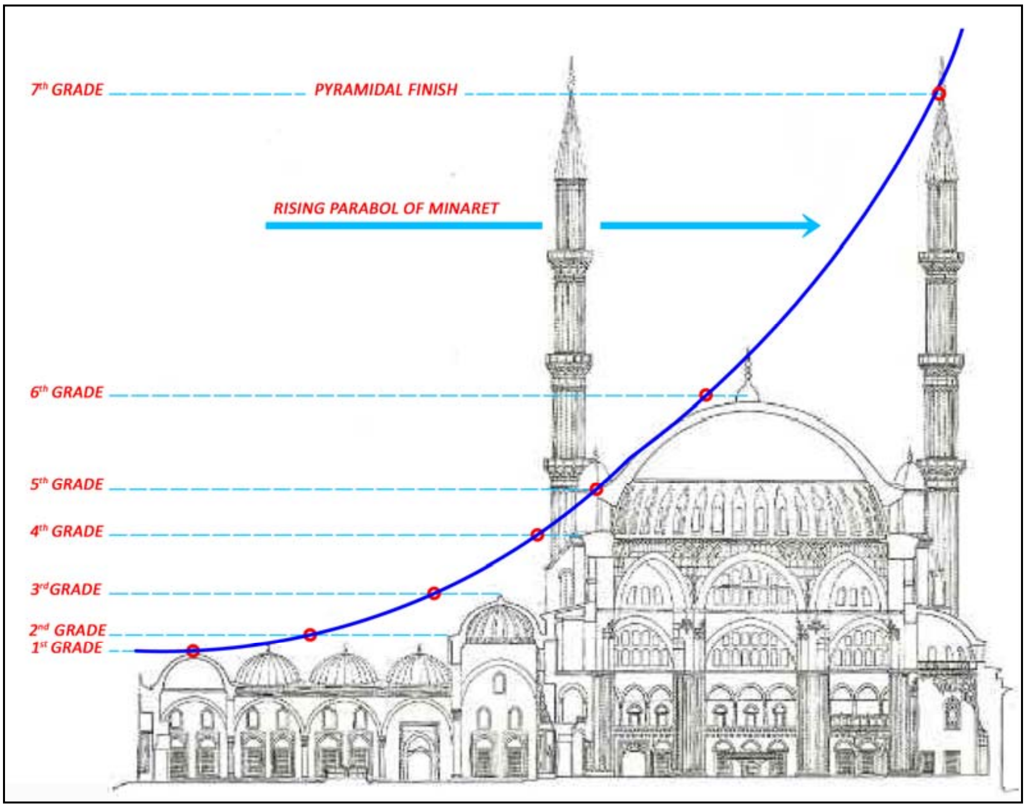

Figure 6. Hierarchy study on longitudinal section of the Selimiye Mosque

Grades (points), which were designated on longitudinal sections of the Mosque, were measured by using the "TOPCON GTS-229 Total station" range finding device. Measuring method is based on the principle of finding heights of objects, whose distance is unknown or could not be measured, by means of trigonometric measurements.

Height differences, which were trigonometrically determined, are randomly and systematically affected by many parameters. Accuracies of these height differences can be defined with error propagation rule. Measured length and errors that influence vertical angles in trigonometric levelling are completely randomly distributed and they are not related. Simultaneously measuring vertical angles and distance by electronic range finders, electronic tacheometers or measuring devices like total station was proved (Soycan and Topbaş, 2002). In this regard, efficiency of trigonometric levelling method has increased and it was scientifically and empirically proved that 5-10 mm/km accuracy can be obtained depending on distance of observation

4.1. CALCULATING HEIGHTS OF OBJECTS, WHOSE DISTANCE IS UNKNOWN OR CANNOT BE MEASURED

Lateral distance can be estimated by means of vertical triangles. After setting up a device on a point A, a point B is ascertained on AK direction (Figure 7). From these two points, zenith angles of Z1 and Z2 and the distance AB=U1 are measured. Since projection of minaret finial cannot be reached, U2 could not be measured and it should be found via calculation.

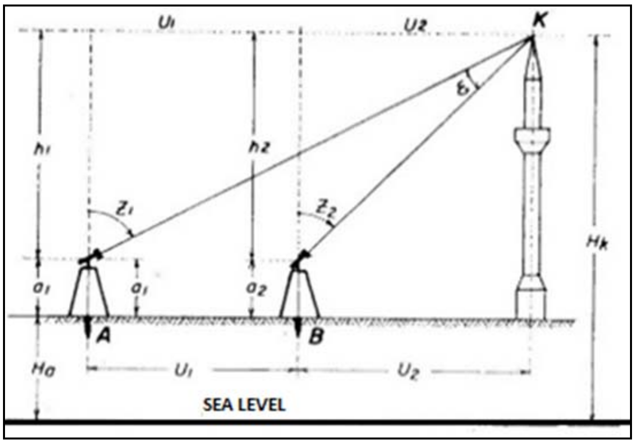

Figure 7. Calculation of minaret distance from vertical triangles (Songu, 1981)

As seen in Figure 7,

Hk = Ha+a1+h1=Ha+a1+(U1+U2)cot Z1 (2)
Hk = Hb+a2+h2=Hb+a2+u2x cot Z2

U2 can be calculated from these two formulas. Since both formulas provide Hk, the following can be written:

Ha+a1+(u1+u2)x cot Z1=Hb+a2+u2 xcotz2

Here, if U2 were to be calculated, the following formula is found:

$$U2 = \frac{(Hb - Ha) + (a2 - a1) - u1 . \cot z1}{\cot z1 - \cot z2} \qquad (1)$$

After calculating U2, Hk can be calculated twice and in a controlled way by means of (2) formulas.

4.2. CALCULATION OF HEIGHT OF OBJECTS, WHOSE DISTANCE IS KNOWN

If the distance **u** between the point A, where a device will be set up, and object, whose distance will be estimated, is known and difference of height y between that object and ground is intended to be measured, the following formula is used.

y=u(cotZ1-cotZ2) (3)

Even if the point where device will be set up is high, formula still remains the same (Figure 8). Because, since sign of **cotZ2** is (-), sign of **h2** becomes (+).

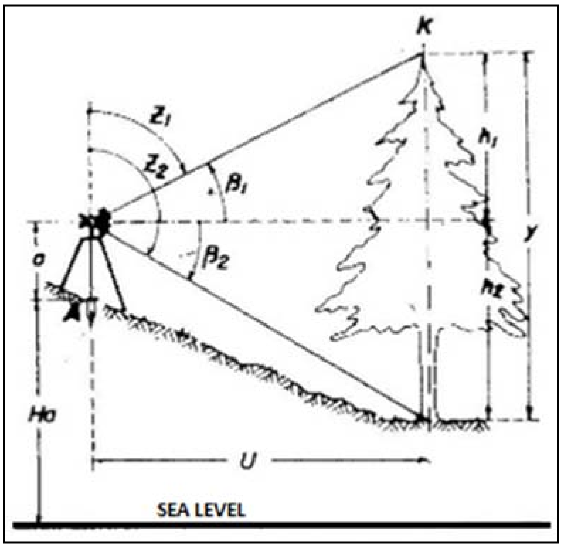

Figure 8. Calculation of height of an object, whose distance can be measured (Songu, 1981)

Hierarchy heights of the Selimiye Mosque were measured by means of the formulas (1) and (2) used in calculation of heights of objects, whose distance is unknown or cannot be measured. In measurements, the points P1 and P2 were marked on ground in an appropriate way that grade points of the Selimiye Mosque can be seen. Local elevations were given to the points P1 and P2. Grade heights were found by taking the Narthex ground (the point which centers longitudinal section) as zero elevation on inner court of the Selimiye Mosque. Measurement values are specified in Table 1.

Table 1. *Measurement values of the Selimiye Mosque Grades* (Kısa Ovalı et al., 2013)

Waited Point	**Observed Point**	**Z (Vertical Angle)**	**S (Distance)**	**Δh (height difference)**	**Explanation (i=device height, a=mark height)**
P1	7th grade P2	71,3950	10,996	0,166	i=1,54 a=1,50
P2	7th grade	69,6700			i=1,45 a=1,50
P1	2nd grade 3rd grade 4th grade 5th grade P2	70,8386 72,7846 68,8278 64,8176	5,729 38,154	0,126	i=1,395
P2	2nd grade 3rd grade 4th grade 5th grade	66,8146 68,1748 65,5880 61,4668			i=1,515
P1	6th grade P2	80,4674	10,788	0,126	i=1,575
P2	6th grade P1	78,9768	10,788	-0,209	i=1,515

The following height values were calculated regarding grades of the Selimiye Mosque by utilizing the formulas (1) and (2). Grade codings and “**H**” codings are represented with the same figures.

H2=16.86 m

H3=20.95 m
H4=27.97 m
H5=33.20 m
H6=42.68 m
H7=79.69 m

Height value regarding the first grade of the Selimiye Mosque was calculated by means of the formula (3), which is used for calculating height of objects whose distance can be measured, as follows within the context of data in Table 2.
H1=14.99 m

Table 2. *Measurement values of the Selimiye Mosque Grades*

Observed Point	Z (Vertical Angle)	S (Distance)
1	100.7004	28.72
2	69.9332	28.72

In this respect, grade heights that will underlie search for mathematical formulas in dimensional hierarchy of the Selimiye Mosque are as follows:

1st Grade, H1=14.99 m
2nd Grade, H2=16.86 m
3rd Grade, H3=20.95 m
4th Grade, H4=27.97 m
5th Grade, H5=33.20 m
6th Grade, H6=42.68 m
7th Grade, H7=79.69 m

4.3. SEARCH FOR MATHEMATICAL FUNCTION IN DIMENSIONAL HIERARCHY

As a result of measurement practices performed, obtained data are point values and there is no certain function definition among these data. Data in measurements performed were obtained as point pairs in the form of (x1, y1) , . . . ,(xn , yn). We intended to be able to determine whether there is a f(x) function correlation among these data in a way that j = 1 , . . . , f(xj) ≈ yj for n.

In the context of above mentioned explanation, data obtained as a result of measurements performed for the Selimiye Mosque as subject of this study is as follows (Table 3), (Figure 9):

Table 3. *Values for the Selimiye Mosque* (Kısa Ovalı et al., 2013)

Point	x (m)	y (m)
1	6,67	14,99
2	36,27	16,86
3	41,60	20,95
4	52,80	27,97
5	54,94	33,20
6	70,67	42,68
7	91,47	79,69

It was determined that the first situation where value of difference of least squares, R, was the smallest in other words polynomial display was the most suitable function for the Selimiye Mosque:

y = 0,0091x2-0.2466x+15.768

As a result of these mathematical function calculations, it was observed that static creativity, which was configured by Architect Sinan in order to carry main dome in a non-problematic way, had a polynomial characteristic and it created aesthetic proportions in outer structural configuration of the Mosque.

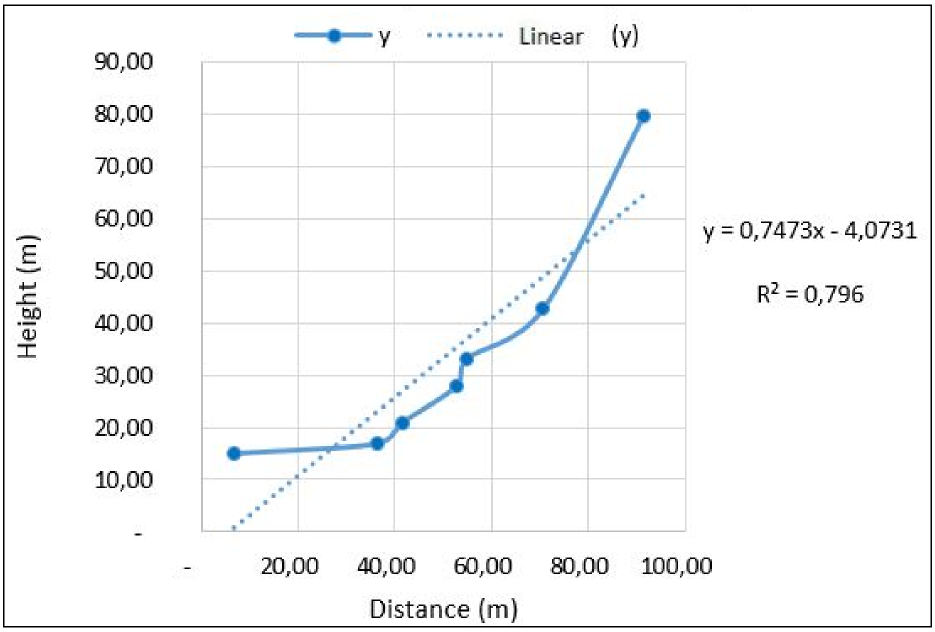

Figure 9. Function Graph of Selimiye Mosque

5. CONCLUSION

It is seen in study carried out within the research context that the Selimiye Mosque has spatial configuration that can respond to many factors which shape design such as social structure, culture, function, time, technology, authority and ideology. In this respect, results which were achieved via researching mathematical trace of dimensional hierarchy of this monumental building which appears in the World Heritage List are being presented to scientific platform. In this study, the presence of function was examined within the context of height values acquired by measuring points specified on the mosque section within the framework of dimensional hierarchy.

As a result of measurement studies carried out, the method for determining a constant function which passes by obtained points as close as possible depending on obtained data being point values was determined and in this regard, function value was found. In this respect;

• It was determined that the most suitable function for the Selimiye Mosque was polynomial display with;

y = 0,0091x2-0.2466x+15.768

As a result of these mathematical calculations, it was observed that structural hierarchy which was set up by Sinan in the Selimiye Mosque had a curvilinear (polynomial) character and this formed an aesthetic transition in the Mosque’s mass.

Function value regarding polynomial curve obtained specific to the Selimiye Mosque can be used for aesthetic configuration of mass balance in newly created urban spheres (major settlement regions) or in complex structures. Also, this hierarchy curve can be used for revision of master plans or preparing new master plans in terms of preserving Edirne skyline. Distances and heights of other buildings that will be constructed around Selimiye, which will be defined as center, can be identified with polynomial curve resulting from own values of Selimiye.

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